Proof Strategies

On a single page here:

http://theaetetus.tamu.edu/logic/lecture/SL-Strategies.pdf

Atom

- Assume its negation for RAA, then seek a contradiction; by means of the contradiction, discharge the assumption by RAA, obtaining the desired atom.

$(\sim P \rightarrow Q), \sim Q \vdash P$

1	(1) $(\sim P \rightarrow Q)$	A	
2	(2) $\sim Q$	A	$\vdash P$
3	(3) $\sim P$	A (for RAA)	
1,3	(4) Q	1,3 $\rightarrow$E	
1,2	(5) P	2,4 RAA (3)	

Negation

- Assume the unnegated WFF for RAA, then seek a contradiction, discharge the assumption, obtaining the desired negation.

~(P v Q) |- ~P

1	(1) ~(P v Q)	A	|- ~P
2	(2) P	A (for RAA)	
2	(3) (P v Q)	2 vI	
1	(4) ~P	1,3 RAA (2)	

Conditional

- Assume its antecedent for →I, then establish its consequent; discharge the assumed antecedent by →I, obtaining the desired conditional.

(P v Q) |- (~P → Q)

1	(1) (P v Q)	A	|- (~P → Q)
2	(2) ~P	A (for →I)	
1,2	(3) Q	1,2 vE	
1	(4) (~P → Q)	3 →I (2)	

Conjunctions

- Obtain its conjuncts separately, then use &I to obtain the desired conjunction.

(P → Q), (P & R) |- (Q & R)

1	(1) (P → Q)	A	
2	(2) (P & R)	A	|- (Q & R)
2	(3) R	2 &E	
2	(4) P	2 &E	
1,2	(5) Q	1,4 →E	
1,2	(6) (Q & R)	3,5 &I	

Biconditional

- Obtain the two required conditionals separately, then use ↔I to obtain the desired biconditional.

(P ↔ Q) |- (Q ↔ P)

1	(1) (P ↔ Q)	A	|- (Q ↔ P)
1	(2) (Q → P)	1 ↔E	
1	(3) (P → Q)	1 ↔E	
1	(4) (Q ↔ P)	2,3 ↔I	

Disjunction

- Assume its negation for RAA, then perform the following three-line procedure:

  I. Assume one of the disjuncts of the disjunction
  II. Use ∨I to obtain the disjunction
  III. Use RAA to discharge the assumed disjunct

  (Bear in mind that this procedure can be repeated, starting with the other disjunct.)

~(~P & ~Q) |- (P v Q)

1	(1) ~(~P & ~Q)	A	|- (P v Q)
2	**(2) ~(P v Q)**	**A (for RAA)**	
3	(3) P	A (for RAA)	
3	(4) (P v Q)	3 vI	
2	(5) ~P	2,4 RAA (3)	
6	(6) Q	A (for RAA)	
6	(7) (P v Q)	6 vI	
2	(8) ~Q	2,7 RAA (6)	
2	(9) (~P & ~Q)	5,8 &I	
1	(10) (P v Q)	1,9 RAA (2)	